Role of Radiative Transport in the Propagation of Laser Supported Combustion Waves

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We report results of an investigation into the roles played by thermal and radiative transport in the propagation of Laser Supported Combustion (LSC) waves. We replace the volume radiation losses assumed by Raizer with detailed treatment of radiation in 19 frequency groups, the flux of each being calculated from the temperature in discrete cells along the LSC wave axis and allowed to deposit in other cells according to temperature dependent absorption coefficients for air. Calculations of LSC wave structure and propagation velocity as a function of laser intensity are compatible with available experimental data.

Nomenclature

α = average cold air absorption length of thermal radiation, cm^{-1}
c = speed of light, cm sec^{-1}
C_p = specific heat at constant pressure, erg g^{-1} K^{-1}
f = ratio of thermal conductivity to specific heat at constant pressure, g cm^{-1} sec^{-1}
h = specific enthalpy, erg g^{-1}
$\hbar$ = Planck constant/$2\pi(1.054\times10^{-27}$ erg sec)
$h_{i(a)}$ = specific enthalpy evaluated at ignition (ambient) temperature, ergs g^{-1}
K = absorption coefficient of laser flux, cm^{-1}
k = Boltzmann constant, 1.380×10^{-16} erg K^{-1}
K_ν' = absorption coefficient corrected for induced emission at frequency ν, cm^{-1}
λ = thermal conduction coefficient, erg cm^{-1} K^{-1} sec^{-1}
λ_r = radiation conduction coefficient, erg cm^{-1} K^{-1} sec^{-1}
ν = photon frequency, sec^{-1}
ρ_o = density of ambient air, g cm^{-3}
S = laser flux, erg sec^{-1} cm^{-2}
T = temperature, K
ϕ = thermal radiative flux divergence, erg sec^{-1} cm^{-3}
U_ν = radiative energy density per unit frequency arriving from all parts of the LSC wave, erg cm^{-3} sec
$U_{\nu p}$ = equilibrium radiative energy density per unit frequency, erg cm^{-3} sec
V_o = velocity of LSC wave, cm sec^{-1}
x_o = length of LSC wave for which $h \geq h_i$, cm

I. Introduction

AIR plasmas sustained by laser fluxes greater than 10^4w/cm^2 have been observed to propagate up the laser beam at subsonic[1,2] and supersonic[3] velocities. Raizer[4] recognized that these phenomena were analogous to the burning of a combustible mixture, in which energy transfer from regions which have undergone exothermic reaction heats adjacent regions above the ignition temperature at which the reaction proceeds rapidly, enabling the reaction to propagate through the mixture. In the present case, the temperature at which the air becomes ionized and opaque to the laser radiation is analogous to the ignition temperature, and laser deposition analogous to energy release via exothermic reaction. Two modes of energy transfer are possible: one supersonic, in which rapid energy deposition leads to the shock-heating of adjacent regions to the ignition temperature; the other subsonic, in which thermal or radiative transport heats adjacent regions. These modes have been called laser supported detonation (LSD) and combustion (LSC) waves, respectively, as they are analogous to the detonation and combustion of chemical explosives. In the present paper, we concentrate on the LSC mode of propagation, which to date has received little theoretical attention. Raizer[4] invoked thermal conduction as the primary mechanism for energy transfer from the advancing LSC wave into the cold air preceding it, though he recognized that reabsorption of thermal radiation from the hot plasma core could also play a role. The hot LSC wave core of temperature $\simeq 2$ev, being optically thin to its own radiation, represents a volume source of such radiation, so that for beam diameters greater than those ($\leqq$1mm) treated by Raizer,[4] radiative transport may be the dominant propagation mechanism.

In this paper, a model is presented which includes the mechanism of radiative transfer into the ambient air, so that its role relative to thermal conduction can be established. It is found that only with the inclusion of this effect can the propagation of LSC waves maintained by large beams be adequately described, and that the results are consistent with available experimental data.

II. Presentation of Model

We assume a one-dimensional model for an LSC wave, the validity of which is discussed later. In a coordinate frame translating with a steady-state LSC wave of speed V_o, the air flow is taken to be in the positive x-direction and the equation for the conservation of energy which balances convective losses against the net energy gain by thermal conduction, radiation, and laser absorption is

$$\rho_o V_o \frac{dh}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \phi + KS \tag{1}$$

where

$$\phi = c\int_o^\infty d\nu\, K_\nu'(U_{\nu p} - U_\nu) \tag{2}$$

is the continuity equation for the thermal radiation[5] and

$$KS = -dS/dx \tag{3}$$

accounts for the attenuation of the laser beam in the plasma. Because all flow velocities are subsonic, it is assumed that the pressure is everywhere constant; hence the momentum equation involving the pressure is not required.

In contrast to models for which the radiative flux divergence ϕ is treated only as a point loss,[6] the inclusion of ϕ in this

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model through Eq. (2), which couples each point of the LSC wave with all other points via the interaction with its own radiation field, necessarily increases the nonlinearity of the problem.

Nevertheless, these equations can be solved by iteration, resulting in a self-consistent solution between the temperature distribution $T(x)$ and its corresponding radiative flux divergence field $\phi(x)$. This is done by making three assumptions. First, the absorption coefficient $K(T)$ for laser radiation is given by

$$K(T) = \begin{cases} K \simeq 0.6 \text{ cm}^{-1}; & T > T_i \simeq 1 \text{ ev} \\ 0; & T < T_i \end{cases} \tag{4}$$

second, the ratio of the thermal conductivity $\lambda(T)$ to the specific heat at constant pressure, $C_p(T) = dh/dT$, is a constant

$$f \equiv \lambda(T)/C_p(T) \simeq 1.0 \times 10^{-3} \text{ g/cm sec} \tag{5}$$

and third, ϕ can be expressed in terms of the independent variable x of Eq. (1) in the form.

$$\phi(x) = \begin{cases} -\phi_A e^{\alpha x}; & T \leqq T_i, \quad x \leqq 0 \\ \phi_B; & T > T_i, \quad 0 < x < x_o \\ 0; & T \leqq T_i, \quad x \geqq x_o \end{cases} \tag{6}$$

The last assumption expresses the fact that the hot plasma core, assumed transparent to its own radiation, results in a volume loss of energy via photons which are strongly absorbed over a length α^{-1} in the cold air preceding the advancing LSC wave. These assumptions allow a continuous, analytic solution of Eq. (1) given by

$$h(x) = \begin{cases} h_a + (h_i - h_a - \phi_A')\, e^{(\rho_o V_o/f)x} + \phi_A' e^{\alpha x}, & x \leqq 0 \\ h_i + S_o/\rho_o V_o + \{[S_o(e^{-Kx_o} - 1) + \phi_\beta x_o]/\rho_o V_o\}\, e^{(\rho_o V_o/f)(x - x_o)} - (S_o e^{-Kx} + \phi_B x)/\rho_o V_o, & 0 \leqq x \leqq x_o \\ h_i, & x \geqq x_o \end{cases} \tag{7}$$

where

$$\phi_A' = \phi_A/\alpha\rho_o V_o$$

$$v_o = \frac{(\phi_A/\alpha\rho_o) + [(\phi_A/\alpha\rho_o)^2 + 4f(h_i - h_a)(KS_o - \phi_B - \phi_A)/\rho_o{}^2]^{1/2}}{2(h_i - h_a)} \tag{8}$$

and

$$1 - e^{-Kx_o} = \phi_B x_o/S_o \tag{9}$$

Then, having solved analytically for $h(x)$, the temperature profile $T(x)$ can be determined knowing the thermodynamic relation $h(T)$, an analytic approximation of which is shown in Fig. 1.[5] Thus, associated with every function $\phi(x)$ delineated by the parameters ϕ_A, ϕ_B, and α is a unique temperature distribution $T(x)$.

The process of iteration is then begun by arbitrarily selecting a set of numbers ϕ_A, ϕ_B, and α [that is, assuming a solution for $\phi(x)$ called ϕ_{analytic}], determining the associated temperature profile $T(x)$ as outlined above, and then using $T(x)$ to solve Eq. (2) numerically for the resulting thermal radiative flux divergence field $\phi(x)$, called $\phi_{\text{numerical}}$ as discussed below. Then,

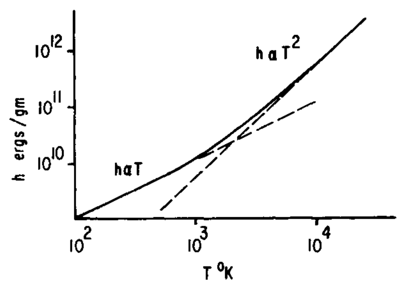

Fig. 1 Specific enthalpy h as a function of temperature T for air, as used in calculations.

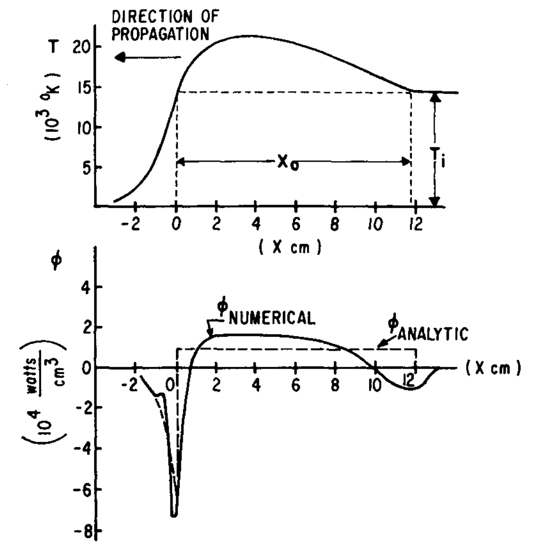

Fig. 2 Self-consistent numerical solution for laser flux of 10^5 w/cm^2: top-temperature profile based on ϕ analytic; bottom-$\phi_{\text{numerical}}$ calculated numerically from temperature profile.

ϕ_{analytic} is compared with $\phi_{\text{numerical}}$ and revised (by changing ϕ_A, ϕ_B, or α) until both functions agree; that is until

$$\int_{-\infty}^{x} \phi(x')_{\text{analytic}}\, dx' \to \int_{-\infty}^{x} \phi(x')_{\text{numerical}}\, dx' \quad \text{for all } x \tag{10}$$

When this is achieved, then a self consistent solution to Eqs. (1–3) is produced subject to the plausible assumptions of Eqs. (4) and (5).

The numerical solution of Eq. (2) is performed by a computer program in which the radiative fluxes in 19 frequency groups emitted from one cell are allowed to deposit energy in all other cells according to frequency dependent absorption coefficients which account for all significant free-free, free-bound, and bound-bound processes arising from molecular, atomic, and ionized combinations of nitrogen and oxygen found in dry air.

A typical self-consistent numerical calculation for $\phi(x)$ is presented in Fig. 2 along with the assumed functions $\phi(x)_{\text{analytic}}$ and $T(x)$ corresponding to a flux of 1×10^5 w/cm^2 from a CO_2 laser. Here, the regions of emission and absorption of thermal radiation are clearly seen.

It is of interest to compare the results of these calculations with those arising from the use of the diffusion approximation which is valid for a quasi-isotropic radiation field. In this case, Eq. (2) would be approximated by[5]

$$\phi \simeq -d/dx[\lambda_r(T)\, dT/dx] \tag{11}$$

An analysis of the spectral contributions to the frontal absorption spike given below reveals that 75% of the absorbed energy lies in the energy range 14.5–20.0 ev over which the absorption coefficient K_ν' is independent of frequency to an excellent approximation. The radiation conduction coefficient $\lambda_r(T)$ for photons in this energy range turns out to be[5]

$$\lambda_r = (k^4 T_3/3\pi^2 K_\nu' c^2 \hbar^3)\, e^{-u}(u^4 + 4u^3 + 12u^2 + 24u + 24)\Big]_{u_2}^{u_1} \tag{12}$$

where $u_j = h\nu_j/kT$. If the diffusion approximation is valid, then ϕ calculated by Eq. (11) should reproduce the absorption spike calculated numerically from Eq. (2) to within 75%. The results of such a calculation yield values for ϕ which are at least a factor of 40 smaller than those calculated numerically. The reason for this huge discrepancy is that the radiation field is strongly anisotropic, thereby making the diffusion approximation of Eq. (11) invalid. Although the diffusion approximation becomes better as x becomes more negative, owing to the increasingly isotropic temperature field, it is important to note that it cannot be used in any realistic LSC wave model since it grossly miscalculates the thermal radiative flux (to which the propagation velocity is proportional) in the region where most of the emitted radiation is reabsorbed.

A more suitable analytic approximation to ϕ according to our calculations would be one in which essentially half of the absorbed laser power is remitted in the forward direction and absorbed exponentially in the cold air. That is, with $S_o(1-e^{-KX_o})/2 = \phi_B x_o/2 \simeq \phi_A/\alpha$

$$\phi(x) = -S_o[(1-e^{-Kx_o})/2]\alpha e^{\alpha x}, \quad x < 0 \tag{13}$$

III. Theoretical Conclusions

A. Discussion of the Energy Transport Mechanism

Because 19 frequency groups have been used to establish the self-consistent solutions, it is possible to determine the relative contributions of each group to the total power deposition in front of the LSC wave. As shown in Fig. 3, the radiation in the frequency group 14.5–20.0 ev is most responsible for the radiative energy deposition and far exceeds that by thermal conduction which deposits energy at the rate $\lambda d^2T/dX^2 \simeq 100$ w/cm^3. According to the self consistent solution of Fig. 2, most of the radiative absorption occurs in the temperature range 5000–14,000 K where atomic nitrogen and oxygen are the dominant species. Since very little of the absorption (< 10%) is due to photons with energies less than the 14.5 ev ionization potential of nitrogen, it is clear that the absorption spike is due primarily to the photo-ionization of atomic nitrogen and, to a smaller degree, atomic oxygen.

In a similar manner, the emission spectrum can be analyzed by frequency group as shown in Fig. 4. Again, most of the emission occurs within the energy range 14.5–20.0 ev at temperatures between 14,000–22,000 K, agreeing with recent spectroscopic and electron density measurements.[2] Here, singly ionized nitrogen and oxygen are the predominant constituents; hence, the ultraviolet emission is due mainly to electron recombination with N^+ and O^+, a process also assumed by Raizer.[4] Being in the energy range 14.5–20.0 ev, this emitted radiation is strongly reabsorbed in the ambient air surrounding the LSC wave and, as explained above, accounts for the large absorption spike of Fig. 2.

B. Discussion of Maintenance Threshold

LSC waves are able to propagate up the laser beam only if the laser flux is greater than a critical value S_c. At this value, the plasma losses are just balanced by the absorption of laser energy and the LSC wave becomes a stationary "plasmatron." According to Eq. (9) as $KS \to KS_c = \phi_B$, the length x_o of the emitting region approaches zero so that the temperature within this region approaches T_i and the thermal radiative flux ϕ_A/α goes to zero. Hence, the propagation velocity V_o of the LSC wave given by Eq. (8) approaches zero. Thus, our model predicts a maintenance threshold S_c of 2.6×10^4 w/cm^2 where K and ϕ_B are evaluated at an ignition temperature of 14,000 K.[4] This threshold is compatible with recent experimental observations which measure S_c to be 3×10^4 w/cm^2 for centimeter beams.[7]

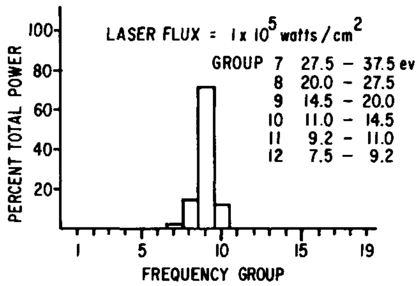

Fig. 3 Percent of total power reabsorbed in ambient air preceding advancing LSC wave in each frequency group.

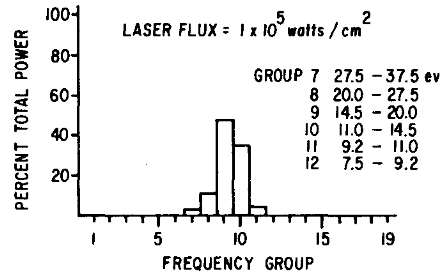

Fig. 4 Percent of total power emitted from LSC in each frequency group.

C. Discussion of Velocity vs Laser Flux Scaling

According to Eq. (8), an LSC wave propagated primarily by thermal conduction will move up the laser beam at a speed V_o which, for laser fluxes well above threshold for maintenance,[4] scales on the square root of the laser flux S_o in the limit of small radiative loss ϕ_B. In this case Eq. (8) becomes

$$V_o \simeq [fKS_o/(h_i - h_a)]^{1/2}/\rho_o \tag{14}$$

However, in the case of a radiation dominated LSC wave, Eq. (8) becomes

$$V_o \simeq (\phi_A/\alpha)/\rho_o(h_i - h_a) \tag{15}$$

Using the same arguments leading to Eq. (13), Eq. (15) becomes

$$V_o \simeq S_o(1-e^{-Kx_o})2\rho_o(h_i - h_a) \tag{16}$$

In Fig. 5 the $V_o(S_o)$ curve corresponding to the self-consistent solution which includes both radiative transport and thermal conduction is compared to the $V_o(S_o)$ curve of Eq. (14) which presupposes only thermal conduction and no radiative losses [curves including volume radiative losses fall below that of Eq. (14)]. It is clear that the dominant propagation mechanism is radiative transport.

It should be pointed out, however, that the dominance of radiative transport over thermal conduction does not occur for all beam radii. For radii ≤ 1 cm, Raizer[4] calculates that the reverse is true owing to the small volume from which the thermal radiation is emitted. Our result should be valid for radii larger than a centimeter.

IV. Comparison with Experiment

When our theory is compared with the experimental observations of Klosterman and Byron,[2] points of agreement as well as

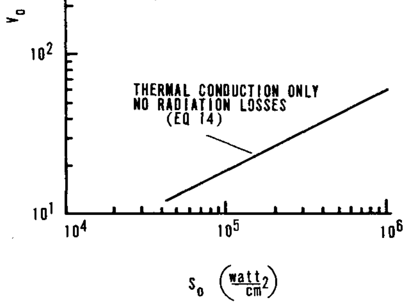

Fig. 5 Theoretical and experimental curves of propagation velocity vs CO_2 laser flux.

disagreement emerge. Included in the points of agreement are: 1) Agreement that the dominant species in the emitting region of the plasma are N^+ and O^+, implying a temperature $\simeq 2$ ev. 2) Agreement on the length scale $\simeq 1$ cm over which the forward heating occurs.

The points of disagreement are: 1) Disagreement with the assumption that the LSC wave, when ignited off a target surface, propagates into undisturbed, ambient air as evidenced by the observation[2] of a shock wave preceding the propagating LSC wave. 2) Disagreement with the assumption that the hydrodynamic flow is one dimensional, as interferograms taken of the LSC wave show clear evidence of radial flow.[2] 3) Disagreement on the magnitude of the propagation velocity as a function of laser flux (See Fig. 5). Thus it appears that our model agrees with experiment in calculating quantities that relate to LSC wave structure but disagrees with those pertaining to its propagation.

However, it is possible to reconcile the above points of discrepancy by noting that Klosterman and Byron ignited LSC waves off a target surface. The rapid heating of the air near the target first by the hot target vapor and then by the absorption of laser energy produces a shock wave owing to the rapid increase in pressure. The existence of a precursor shock wave, which has been observed by Klosterman and Byron, implies that to some degree the air into which the newly formed LSC wave propagates (by thermal conduction and radiative transport) has a gross motion which must be subtracted from the observed, laboratory velocity in order to validly compare theory with experiment.

We hypothesize that this is the reason why the experimental value for the propagation velocity is higher than our calculated value. This is suggested by the fact that the experimental values for the propagation velocity are higher than the maximum allowable velocity V_o^{max} for a 1-D model found by assuming that all the incident laser flux is converted by some extremely efficient energy transport mechanism into the forward heating of the adjacent cold air thereby giving rise to propagation. That is, S_o is equated to $\rho_o V_o^{max}(h_i - h_a)$ to yield the relation

$$V_o^{max} = S_o/\rho_o(h_i - h_a) \qquad (17)$$

which is plotted in Fig. 5. Thus, it would appear that: 1) the assumption of one dimensionality inherent in Eq. (17) has been violated in such a way that the effective density ρ_o has been reduced to yield a larger value for V_o^{max} (a possibility since 2-D flow is observed), or 2) that the influences of the precursor shock wave are present. To evaluate these possibilities, we have performed further calculations of LSC wave propagation in the presence of a fixed boundary using a hydrodynamic computer program. The results of such calculations indicate that the precursor shock wave imparts to the air a velocity at least as large as the velocity with which the LSC wave propagates with respect to the moving air, and that the effect of two-dimensional flow is secondary, tending to reduce the forward propagation velocity rather than increase it. The result of a typical calculation is shown in Fig. 6, where the precursor shock wave, radial flow, and other experimentally observed features[2] are clearly evident. Details of these calculations are to be published separately.[8]

In conclusion, therefore, we believe our model agrees with experiments performed near a target surface if the observed

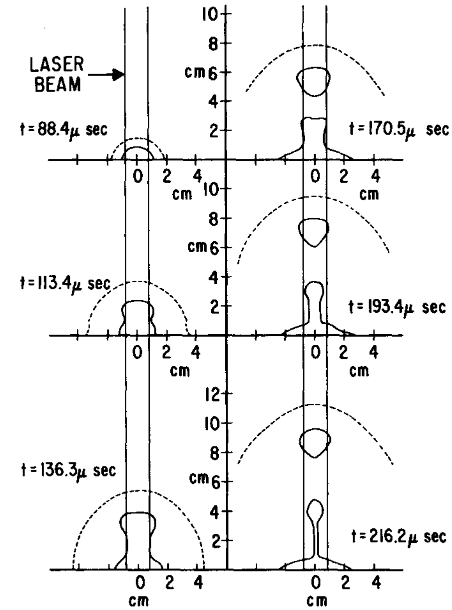

Fig. 6 Hydrodynamic computer calculations showing LSC wave propagation near a target surface (CO_2 laser flux = 1×10^6 w/cm^2, laboratory propagation velocity = 7×10^4 cm/sec).

propagation velocity is correctly interpreted. Far from the target, we believe our model is strictly applicable and in both cases is an accurate assessment of the role played by radiative transport in LSC wave propagation.

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